

KWINANA N.E.



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2
NOVEMBER 2019
MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

R. Govender
Finalise
external
approval
6/11/2019

These marking guidelines consist of 26 pages.
Hierdie nasienriglyne bestaan uit 26 bladsye.

DEPARTMENT OF BASIC
EDUCATION
PRIVATE BAG X895, PRETORIA 0001
2019 -11- 06
APPROVED MARKING GUIDELINE
PUBLIC EXAMINATION

Approved
Choe
2019-11-06

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

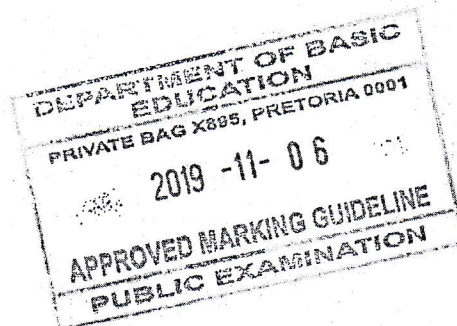
- As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.

Sx

R

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)</i>
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)</i>
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>

*tolerance

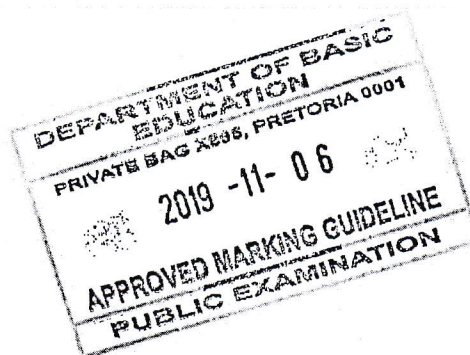


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QUESTION/VRAAG 1

Monthly income (in rands) Maandelikse inkomste (in rand)	9 000	13 500	15 000	16 500	17 000	20 000
Monthly repayment (in rands) Maandelikse paaieiment (in rand)	2 000	3 000	3 500	5 200	5 500	6 000

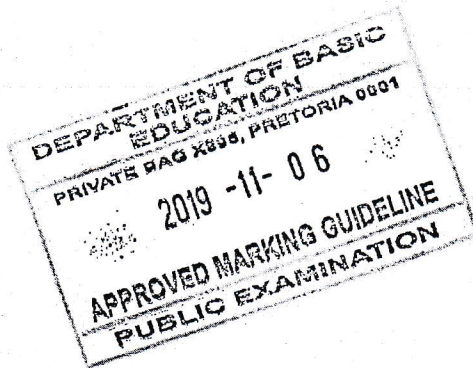
1.1	$a = -1946,875... = -1946,88$ $b = 0,41$ $\hat{y} = -1946,88 + 0,41x$ <i>no penalty for rounding</i> <i>CA eqn</i> <i>and b swapped max 1/3 - answer only wrong a+b c/3</i>	<div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only: Full marks</div>	✓ $a = -1946,88$ ✓ $b = 0,41$ ✓ equation	(3)
1.2	Monthly repayment \approx R3 727,16 Maandelikse paaieiment \approx R3 727,16 (calculator)		✓✓ answer	(2)
	OR $\hat{y} = -1946,88 + 0,41(14000)$ \approx R3 793,12 <i>y \neq -ve ; realistic</i>		✓ substitution ✓ answer	(2)
1.3	$r = 0,946 \dots \approx 0,95$ <i>0,9 ; 0,94 not 1</i>		✓ answer	(1)
1.4	Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. OR D Spandeer nie R9 000 per maand nie, want die punt (18 000 ; 9 000) lê baie ver van die kleinste-kwadrade regressielyn. OF D		✓✓ answer <i>Accuracy</i>	(2)
				[8]



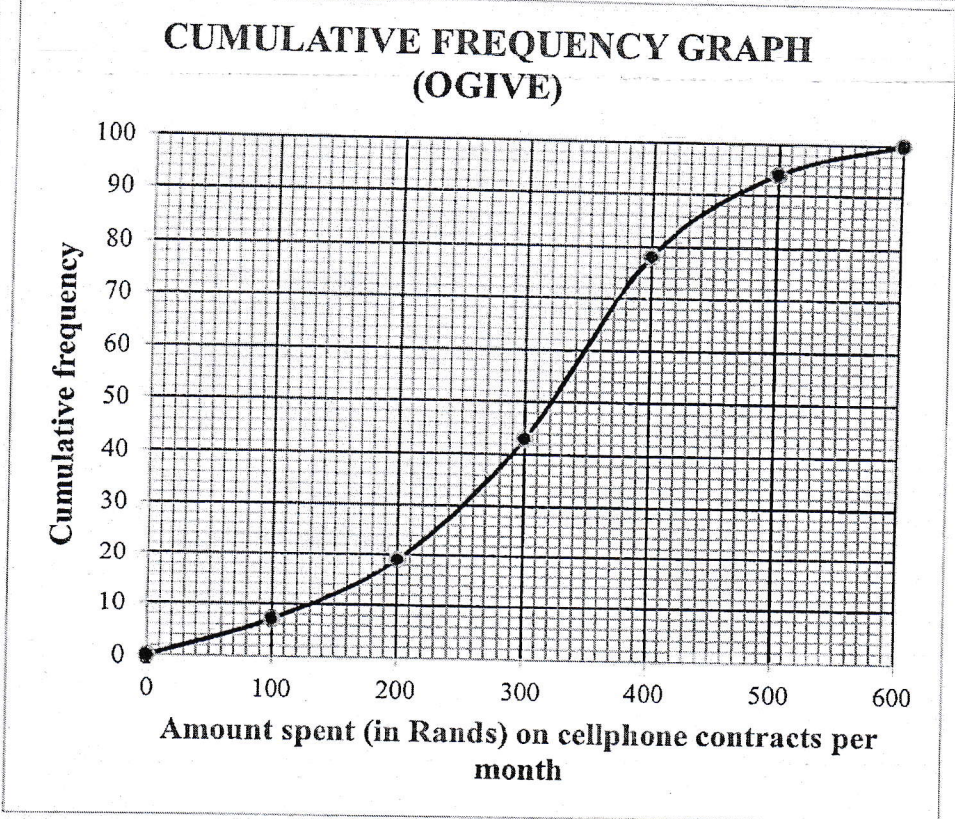
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QUESTION/VRAAG 2

2.1	Number people paid R200 or less = 19 <i>Aantal mense wat R200 of minder betaal het = 19</i>	✓ answer (1)
2.2	$7+12+a+35+b+6=100$ ✓ $a+b=40$ ✓ $a=40-b$ ✓ $a+b=40$ ✓ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$ ✓ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times (40 - b)) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$ ✓ $350 + 1800 + 10000 - 250b + 12250 + 450b + 3300 = 30900$ $200b = 3200$ ✓ <i>no midpoints max 2/5(freq.)</i> $b = 16$ $a = 24$ OR/OF $5a + 9b = 264$ $5a + 5b = 20$ <i>sim. eqns</i> $4b = 64$ ✓ $7+12+a+35+b+6=100$ $b=40-a$ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times (40 - a)) + (550 \times 6)}{100}$ $350 + 1800 + 250a + 12250 + 1800 - 450a = 30900$ $200a = 4800$ $a = 24$ $b = 16$	✓ $\sum x = 100$ ✓ $a = 40 - b$ ✓ $\sum fX$ <i>numerical</i> ✓ $\sum \frac{fX}{n} = 309$ ✓ $200b = 3200$ (5) ✓ $\sum x = 100$ ✓ $b = 40 - a$ ✓ $\sum fX$ ✓ $\sum \frac{fX}{n} = 309$ ✓ $200a = 4800$ (5)
2.3	Modal class/modale klas: $300 < x \leq 400$ <i>between 300 and 400</i> <i>from 300 to 400</i> <i>don't worry about notation</i>	✓ answer (1)



2.4



- ✓ grounded at (0 ; 0)
- ✓ (600 ; 100) (last one)
- ✓ cumulative frequencies for y-coordinates (2 more correct p)
- ✓ smooth shape (no ruler)
- midpts $\frac{3}{4}$ - lose 2nd mark
- lower limits $\frac{3}{4}$

(4)

2.5 Number of people/Aantal mense = 100 - 82 [accept 80 - 84 people]
 18 people paid more than R420 per month/. [accept 16 - 20 people]
 18 mense betaal meer as R420 per maand

Answer only: Full marks

- ✓ 82
- ✓ answer

(2)

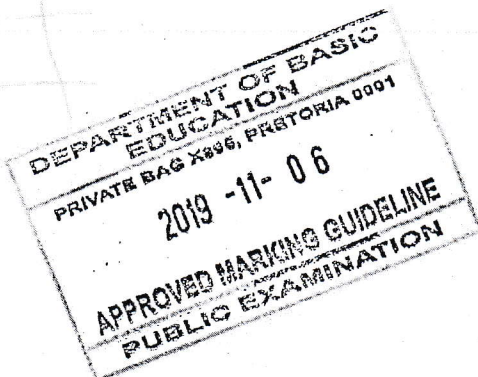
[13]

If ogive not correct go to graph, follow and give full marks

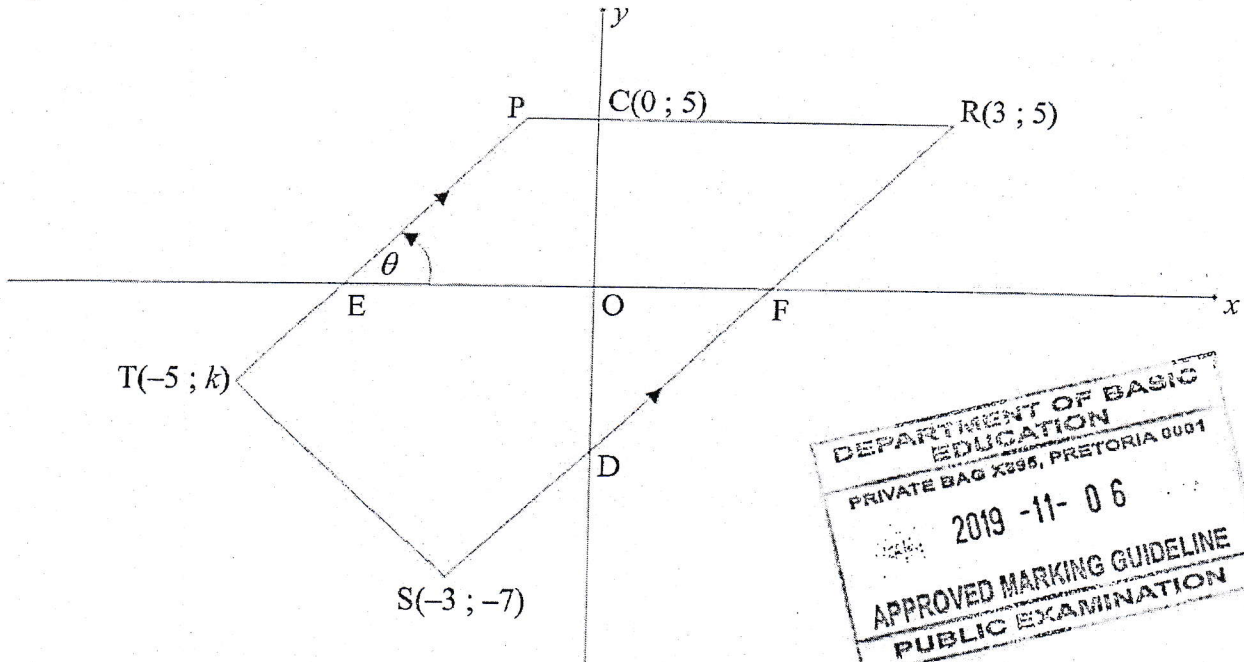
** 16 < answer only < 22 ogive not given*

2.5 no answer but indicated on graph, mark

Upper limit	CF
100	7
200	19
300	43
400	78
500	94
600	100



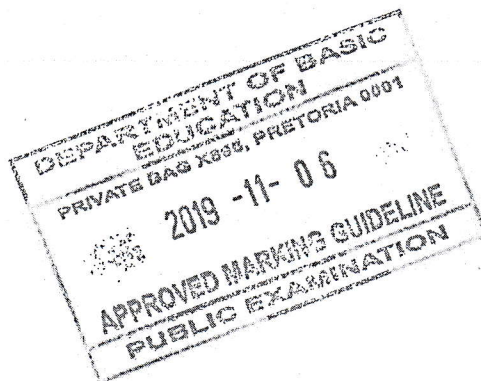
QUESTION/VRAAG 3



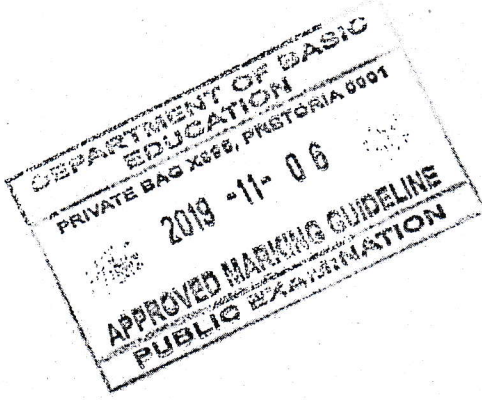
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3.1	Equation of PR: $y = 5$	✓ answer (1)
3.2.1	$m_{RS} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{RS} = \frac{5 - (-7)}{3 - (-3)} = \frac{12}{6} = 2$ <p>if $m = \frac{\Delta x}{\Delta y}$ 1/2</p> <p>Answer only: Full marks</p> $m = \frac{y_2 - y_1}{x_1 - x_2}$ max 1/2	✓ substitution of R & S into gradient formula ✓ answer (2)
3.2.2	$m_{RS} = m_{PT}$ [PT RS] $\tan \theta = 2$ $\theta = 63,43^\circ$ RS ⊥ PT max 1/3 $\tan \theta = 2$ ✓ CA from 3.2.1 $m = 1$ from nowhere max 1/3 $\theta = \tan^{-1}(2)$ ✓	✓ $m_{RS} = m_{PT}$ ✓ $\tan \theta = 2$ ✓ $\theta = 63,43^\circ$ answer only 3/3 (3)
3.2.3	Equation of RS: $y - 5 = 2(x - 3)$ or $y - (-7) = 2(x - (-3))$ or $5 = 2(3) + c$ $y - 5 = 2x - 6$ $y + 7 = 2x + 6$ $c = -1$ $y = 2x - 1$ $y = 2x - 1$ $y = 2x - 1$ $\therefore D(0; -1)$ CA m from 3.2.1 OR/OF $m_{RS} = m_{RD} = m_{DS}$ $2 = \frac{5 - y}{3 - 0} = \frac{y + 7}{0 - (-3)}$ $\therefore y = -1$ $\therefore D(0; -1)$ Assuming D midpt ✓	✓ substitution BD any other point ✓ equation of RS ✓ coordinates of D $-ve m$ $5 = -2(3) + c$ OR $c = 11$ $y = -2x + 11$ $D(0; 11)$ ✓ equating gradients ✓ value of y ✓ coordinates of D (3)

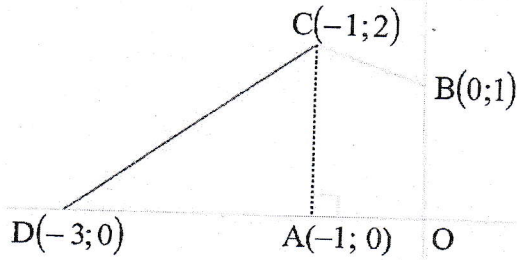
<p>3.3</p>	$ST = 2\sqrt{5} = \sqrt{[-5 - (-3)]^2 + (k - (-7))^2}$ $20 = 4 + (k + 7)^2$ $(k + 7)^2 = 16$ $k + 7 = \pm 4 \quad \checkmark \text{ or just } 4$ $k = -11 \text{ or } k = -3 \text{ don't penalise}$ $\therefore k = -3 \quad \checkmark$ <p>OR</p> $ST = 2\sqrt{5} = \sqrt{[-5 - (-3)]^2 + (k - (-7))^2}$ $20 = 4 + k^2 + 14k + 49$ $k^2 + 14k + 33 = 0$ $(k + 11)(k + 3) = 0$ $k = -11 \text{ or } k = -3$ $\therefore k = -3$	<ul style="list-style-type: none"> ✓ substitute S and T into distance formula ✓ isolate square ✓ square root both sides ✓ answer <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> ✓ substitute S and T into distance formula ✓ standard form ✓ factors ✓ answer <p style="text-align: right;">(4)</p>
<p>3.4</p>	<p>Method: translation T → S:</p> $(x; y) \rightarrow (x + 2; y - 4)$ <p>∴ by symmetry: D → N:</p> $D(0; -1) \rightarrow N(0 + 2; -1 - 4)$ $\therefore N(2; -5)$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px auto;">Answer only: Full marks</div> <p>OR</p> <p>Midpoint of TN = Midpoint of SD</p> $\frac{x + (-5)}{2} = \frac{-3 + 0}{2} \quad \text{and} \quad \frac{y + (-3)}{2} = \frac{-7 + (-1)}{2}$ $x = 2 \quad \text{and} \quad y = -5$ $\therefore N(2; -5)$ <div style="border: 1px solid black; padding: 2px; width: fit-content; margin: 5px auto;">Answer only: Full marks</div>	<ul style="list-style-type: none"> ✓ method ✓ x-coordinate ✓ y-coordinate <p>$N(0 + 2; 11 - 4)$ $(2; 7)$ max 2/3 4th quad</p> <p style="text-align: right;">(3)</p> <ul style="list-style-type: none"> ✓ method: midpoint of diagonals ✓ x-coordinate ✓ y-coordinate <p style="text-align: right;">(3)</p>



<p>4.3</p>	<p> $m_{\text{radius}} = \frac{2-1}{-1-0} \text{ OR } \frac{2-(-\frac{1}{2})}{-1-\frac{3}{2}} \text{ OR } \frac{0-(-\frac{1}{2})}{1-\frac{3}{2}}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y = mx + c$ $y = x + c$ $2 = 1(-1) + c$ $c = 3$ $\therefore y = x + 3$ $y - x = 3$ </p> <p>OR</p> <p> $m_{\text{radius}} = \frac{2-1}{-1-0}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y - y_1 = m(x - x_1)$ $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y - 2 = x + 1$ $\therefore y = x + 3$ $y - x = 3$ </p> <p><i>can stop here</i></p> <p><i>CA from m_{radius} max 2/4 + subst.</i></p> <p><i>$m_r = -1$ m_{tangent} not calculated max 1/4</i></p>	<p> $\checkmark m_{\text{radius}}$ $\checkmark m_{\text{tangent}}$ </p> <p> \checkmark substitute $(-1; 2)$ and m \checkmark simplification (4) </p> <p> $\checkmark m_{\text{radius}}$ $\checkmark m_{\text{tangent}}$ </p> <p> \checkmark substitute $(-1; 2)$ and m \checkmark simplification (4) </p>
<p>4.4</p>	<p> Tangents to circle: $y = x + 3$ and $y = x + 1$ </p> <p> $t > 3$; $t < 1$ </p> <p> $\therefore t > 3$ or $t < 1$ </p> <p> $t > 3$ or $t < 1$ </p> <p> Answers only: Full marks </p>	<p> $\checkmark y = x + 1$ $\checkmark t > 3$ $\checkmark t < 1$ (3) </p>
<p>4.5</p>	<p>Draw rectangle CNED:</p> <p> $C(-1; 2)$ </p> <p> Midpt of DN $(-\frac{7}{4}; \frac{3}{4})$ </p> <p> $\therefore E(-\frac{5}{2}; -\frac{1}{2})$ </p> <p> $D(-3; 0)$ </p> <p> $N(-\frac{1}{2}; \frac{3}{2})$ </p> <p> E </p> <p> OR/OF $D(-3; 0)$ $C \rightarrow N:$ $(x; y) \rightarrow (x + 0,5; y - 0,5)$ $D \rightarrow E:$ $D(x; y) \rightarrow E(x + 0,5; y - 0,5)$ $\therefore E(-3 + 0,5; 0 - 0,5)$ $\therefore E(-2,5; -0,5)$ </p> <p> Answer only: Full marks </p>	<p> \checkmark midpt of DN $\checkmark x$ value $\checkmark y$ value (3) </p> <p> \checkmark coordinates of D $\checkmark x$ value $\checkmark y$ value (3) </p>



4.6



$$\begin{aligned} \text{area of trapezium AOBC} &= \frac{1}{2}(1+2)(1) \quad \checkmark \\ &= 1\frac{1}{2} \text{ square units} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{area of } \triangle ACD &= \frac{1}{2}(2)(2) \quad \checkmark \\ &= 2 \text{ square units} \quad \checkmark \end{aligned}$$

$$\text{area of quadrilateral OBCD} = 3\frac{1}{2} \text{ square units}$$

$$\therefore 2a^2 = \frac{7}{2}$$

$$a^2 = \frac{7}{4}$$

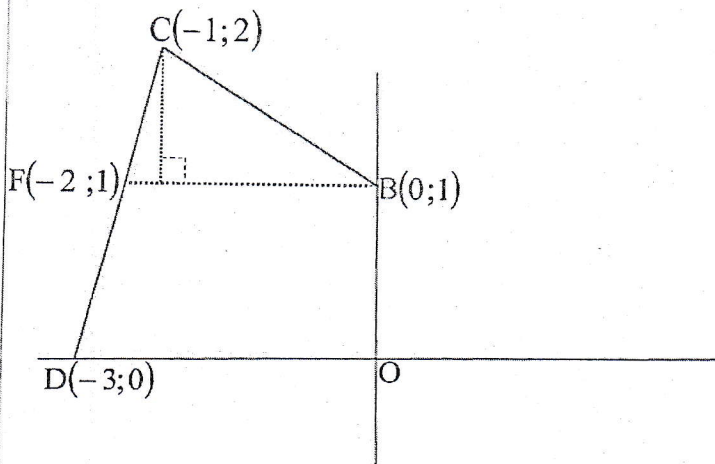
$$a = \frac{\sqrt{7}}{2}$$

OR

- ✓ substitution into area of trapezium form
- ✓ area of trapezium AOBC
- ✓ area of triangle ACD
- ✓ area of OBCD
- ✓ equating area OBCD to $2a^2$

(5)





BM produced cuts the tangent at F.

$$\begin{aligned} \text{area of } \triangle CFB &= \frac{1}{2}(2)(1) \\ &= 1 \text{ square unit} \end{aligned}$$

$$\begin{aligned} \text{area of trapezium BFDO} &= \frac{1}{2}(2 + 3)(1) \\ &= 2\frac{1}{2} \text{ square units} \end{aligned}$$

$$\text{area of quadrilateral OBCD} = 3\frac{1}{2} \text{ square units}$$

$$\therefore 2a^2 = \frac{7}{2}$$

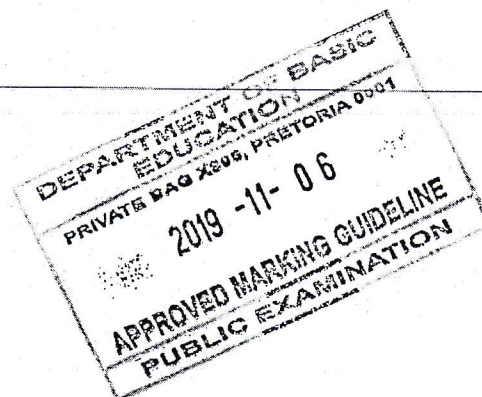
$$a^2 = \frac{7}{4}$$

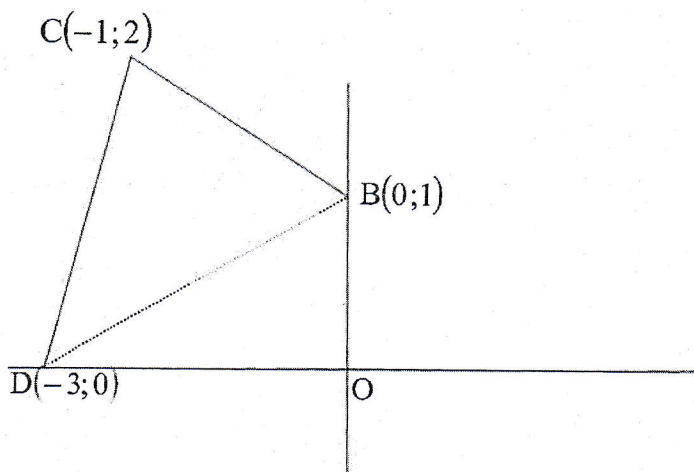
$$a = \frac{\sqrt{7}}{2}$$

OR

- ✓ area of triangle
- ✓ substitution into area of trapezium
- ✓ area of trapezium
- ✓ area of OBCD
- ✓ equating area OBCD to $2a^2$

(5)





Join DB

$$\begin{aligned} \text{area of } \triangle ODB &= \frac{1}{2}(3)(1) \\ &= \frac{3}{2} \text{ square unit} \end{aligned}$$

$$\begin{aligned} \text{area of } \triangle DCB &= \frac{1}{2}(2\sqrt{2})(\sqrt{2}) \\ &= 2 \text{ square unit} \end{aligned}$$

$$\therefore \text{area of OBCD} = \frac{3}{2} + 2 = \text{square units}$$

$$2a^2 = \frac{7}{2}$$

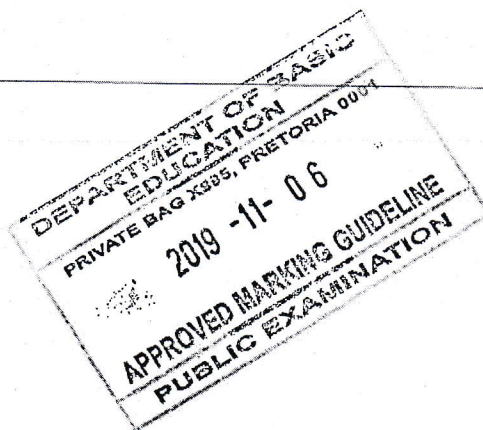
$$a^2 = \frac{7}{4}$$

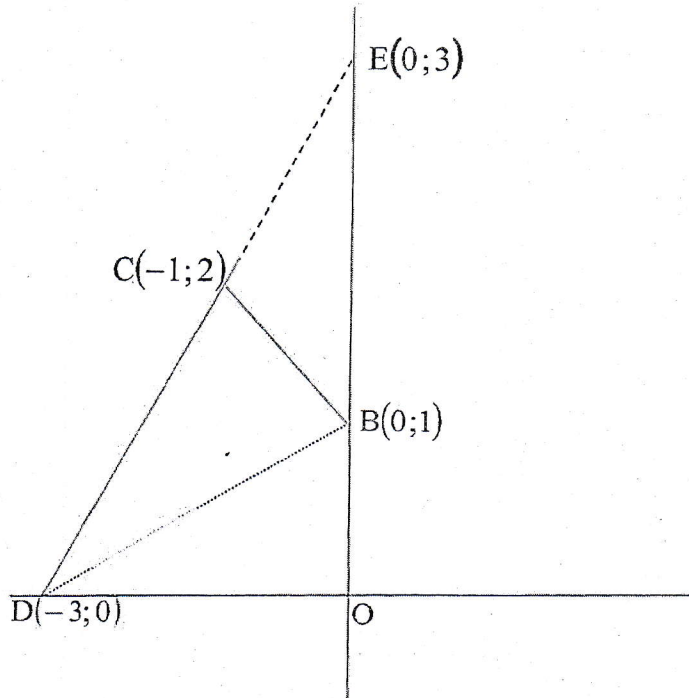
$$a = \frac{\sqrt{7}}{2}$$

OR

- ✓ area of $\triangle ODB$
- ✓ subst into area of $\triangle DCB$
- ✓ area of $\triangle DCB$
- ✓ area of OBCD
- ✓ equating area OBCD to $2a^2$

(5)





Let E be the point of intersection of DC with the positive y-axis.

$$\begin{aligned} \text{area of } \triangle DEO &= \frac{1}{2}(3)(3) \\ &= \frac{9}{2} \text{ square unit} \end{aligned}$$

$$\begin{aligned} \text{area of } \triangle ECB &= \frac{1}{2}(2)(1) \text{ or } \frac{1}{2}(\sqrt{2})(\sqrt{2}) \\ &= 1 \text{ square unit} \end{aligned}$$

$$\text{area of quadrilateral OBCD} = \frac{9}{2} - 1 = 3\frac{1}{2} \text{ square units}$$

$$\therefore 2a^2 = \frac{7}{2}$$

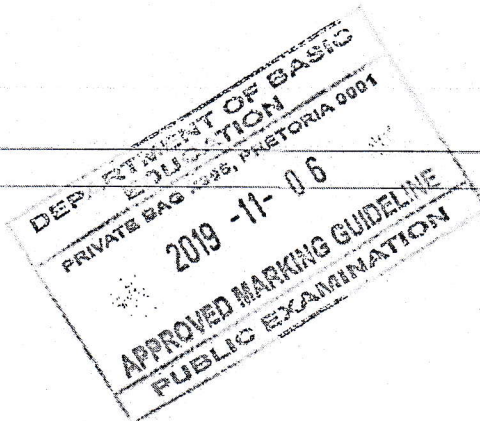
$$a^2 = \frac{7}{4}$$

$$a = \frac{\sqrt{7}}{2}$$

- ✓ area of $\triangle DEO$
- ✓ subst into area of $\triangle ECB$
- ✓ area of $\triangle ECB$
- ✓ area of OBCD
- ✓ equating area OBCD to $2a^2$

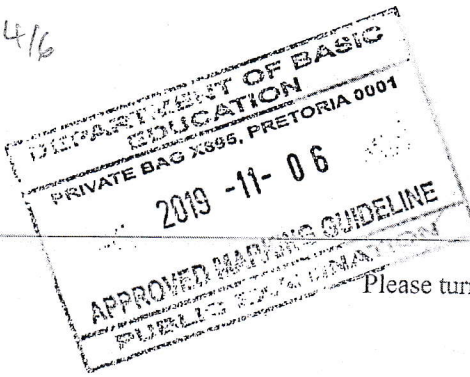
(5)

[20]



QUESTION/VRAAG 5

<p>5.1</p>	$\frac{\sin x}{\cos x \cdot \tan x} + \sin(180^\circ + x) \cos(90^\circ - x)$ $= \frac{\sin x}{\cos x \cdot \frac{\sin x}{\cos x}} + (-\sin x) \sin x$ $= 1 - \sin^2 x \quad \text{CA}$ $= \cos^2 x$ <p><i>Handwritten: $\frac{\sin x}{\cos x} = \tan x$</i></p>	<p>✓ $-\sin x$ ✓ $\sin x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $1 - \sin^2 x$ ✓ $\cos^2 x$</p> <p>(5)</p>
<p>5.2</p>	$\frac{\sin^2 35^\circ - \cos^2 35^\circ}{4 \sin 10^\circ \cos 10^\circ}$ $= \frac{-(\cos^2 35^\circ - \sin^2 35^\circ)}{2(2 \sin 10^\circ \cos 10^\circ)}$ $= \frac{-\cos 70^\circ}{2 \sin 20^\circ}$ $= \frac{-\cos 70^\circ}{2 \cos 70^\circ} \quad \text{OR} = \frac{-\sin 20^\circ}{2 \sin 20^\circ} = -\frac{1}{2}$ <p><i>Handwritten: $-\cos 2(35^\circ)$, $2 \sin 2(10^\circ)$, max 3/4</i></p>	<p>✓ $-(\cos^2 35^\circ - \sin^2 35^\circ)$ ✓ $-\cos 70^\circ$ ✓ $2 \sin 20^\circ$</p> <p>✓ answer</p> <p>(4)</p>
<p>5.3</p>	$2 \sin^2 77^\circ = 2[\sin(90^\circ - 13^\circ)]^2$ $= 2 \cos^2 13^\circ$ $= 2 \cos^2 13^\circ - 1 + 1$ $= \cos 26^\circ + 1$ $= m + 1$ <p>OR</p> $1 - 2 \sin^2 77^\circ = \cos 154^\circ$ $2 \sin^2 77^\circ = 1 - \cos 154^\circ$ $= 1 - (-\cos 26^\circ)$ $= 1 + m$ <p><i>Handwritten: $\cos 26^\circ = m$, $\cos 2(13^\circ) = m$, $2 \cos^2(13^\circ) - 1 = m$, $2 \sin^2 77^\circ - 1 = m$, $2 \sin^2 77^\circ = m + 1$, $2 \sin^2 77^\circ = -1 + \cos 154^\circ$, $2 \sin^2 77^\circ = 1 - \cos 154^\circ$</i></p>	<p>✓ using co-ratio ✓ reduction ✓ $2 \cos^2 13^\circ - 1 = \cos 26^\circ$ ✓ answer</p> <p>(4)</p> <p>✓ $1 - 2 \sin^2 77^\circ = \cos 154^\circ$ ✓ $2 \sin^2 77^\circ = 1 - \cos 154^\circ$ ✓ reduction ✓ answer</p> <p>(4)</p>
<p>5.4.1</p>	$\sin(x + 25^\circ) \cos 15^\circ - \cos(x + 25^\circ) \sin 15^\circ = \tan 165^\circ$ $\sin(x + 25^\circ - 15^\circ) = -0,2679... \text{ OR } -2 + \sqrt{3}$ <p><i>Handwritten: both sides same - 3 marks</i></p> $\sin(x + 10^\circ) = -0,2679... \text{ OR } -2 + \sqrt{3}$ $x + 10^\circ = 195,54^\circ + k \cdot 360^\circ \quad \text{or} \quad x + 10^\circ = 344,46^\circ + k \cdot 360^\circ$ $x = 185,54^\circ + k \cdot 360^\circ; k \in \mathbb{Z} \quad \text{or} \quad x = 334,46^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ <p><i>Handwritten: $-174,46^\circ$, $-25,54^\circ$</i></p> <p>OR/OF</p> <p><i>Handwritten: only 1 solⁿ max 4/6</i></p>	<p>✓ ✓ $\sin(x + 10^\circ)$ ✓ $-0,2679... / -2 + \sqrt{3} / 15,54^\circ$ ✓ $195,54^\circ$ & $344,46^\circ$ ✓ $185,54^\circ$ & $334,46^\circ$ ✓ $+ k \cdot 360^\circ; k \in \mathbb{Z}$</p> <p>(6)</p>



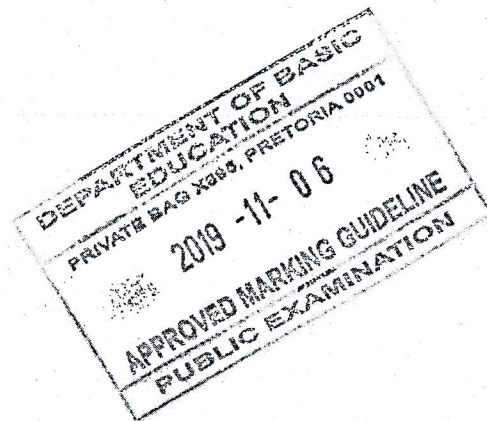
	$\sin(x + 25^\circ) \sin 75^\circ - \cos(x + 25^\circ) \cos 75^\circ = \tan 165^\circ$ $-(\cos(x + 25^\circ) \cos 75^\circ - \sin(x + 25^\circ) \sin 75^\circ) = -0,2679...$ $\cos(x + 100^\circ) = 0,2679...$ $\text{ref. } \angle = 74,4577...^\circ$ $x + 100^\circ = 74,46^\circ + k \cdot 360^\circ \text{ or } x + 100^\circ = 285,54^\circ + k \cdot 360^\circ$ $x = -25,54^\circ + k \cdot 360^\circ; k \in \mathbb{Z} \text{ or } x = 185,54^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$	$\checkmark \checkmark \cos(x + 100^\circ)$ $\checkmark -0,2679...$ $\checkmark 74,46^\circ \text{ \& } 285,54^\circ$ $\checkmark -25,54^\circ \text{ \& } 185,54^\circ$ $\checkmark +k \cdot 360^\circ; k \in \mathbb{Z}$ (6)
5.4.2	$f(x) = \sin(x + 10^\circ)$ $\cos(x + 100^\circ)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 5px 0;"> Answers only: Full marks </div> <p>For minimum value of $\sin x$: $x = 270^\circ$ For minimum value of $\sin(x + 10^\circ)$: $x = 260^\circ$</p>	$\checkmark f(x) = \sin(x + 10^\circ)$ $\checkmark 270^\circ$ $\checkmark \text{ answer}$ (3)
		[22]

$$\sin[(x + 25^\circ) - 15^\circ] = -1$$

$$\sin(x + 10^\circ) = -1$$

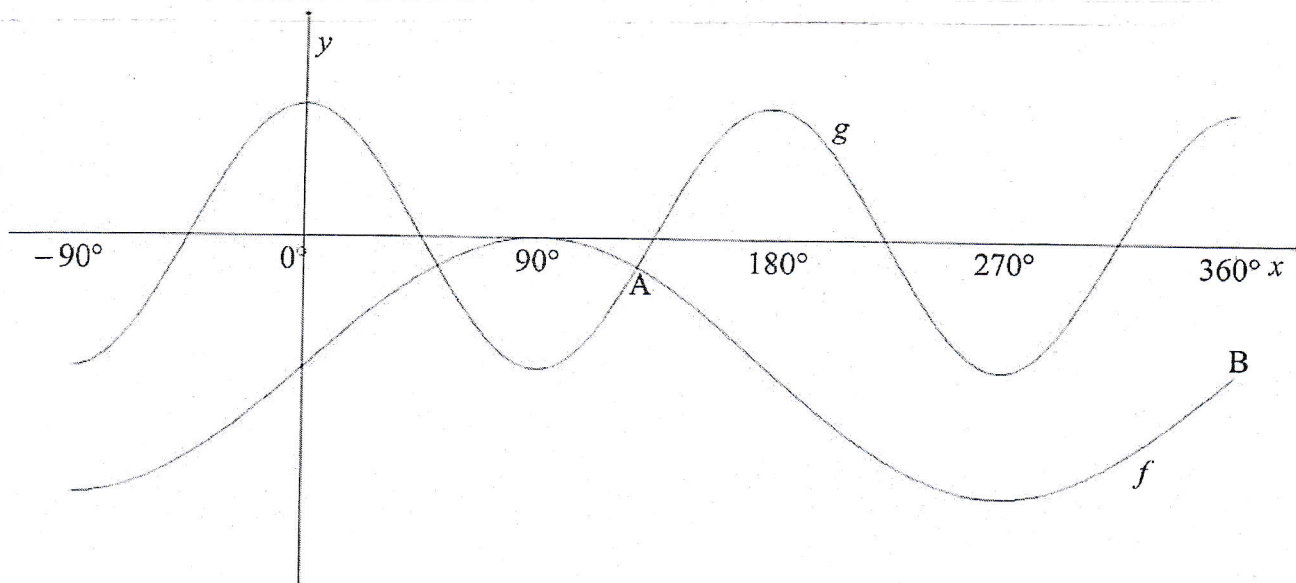
$$x + 10 = 270$$

$$x = 260$$



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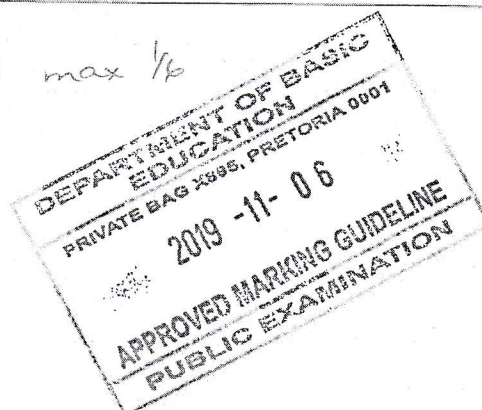
QUESTION/VRAAG 6



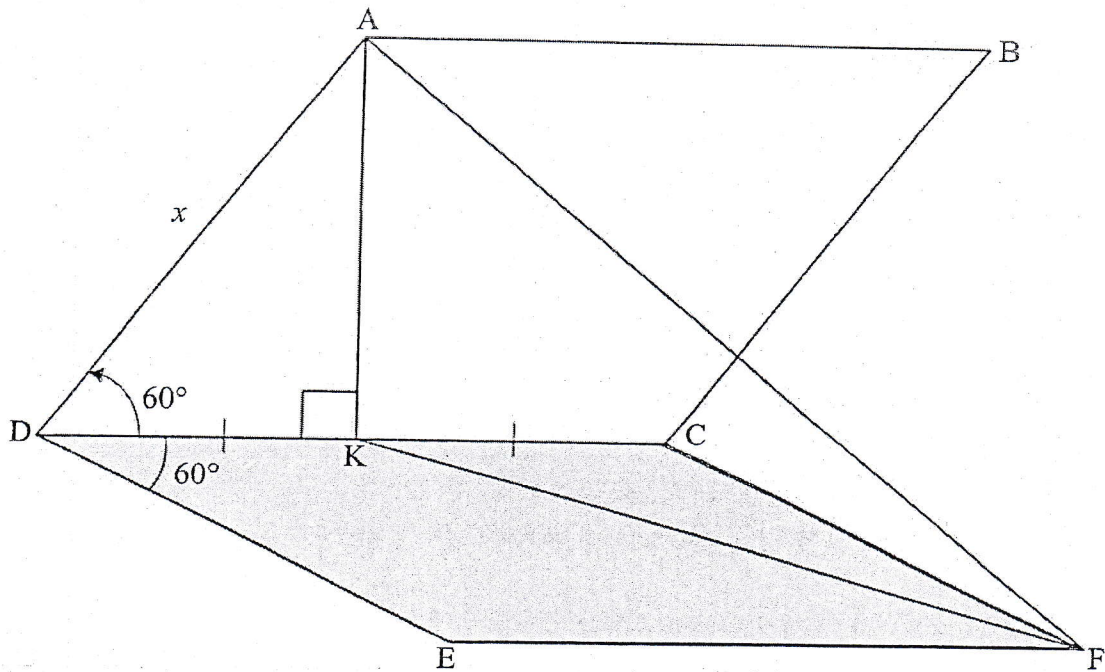
6.1	Range of $f: y \in [-2; 0]$ OR $-2 \leq y \leq 0$ <i>$y \in (-2; 0]$ 1 mark $[-2; 0]$</i>	✓ critical values ✓ notation <i>$x \in [-2; 0]$ interval must be correct</i> (2)
6.2	$x \in (90^\circ; 270^\circ)$ OR $x \in [90^\circ; 270^\circ]$ <i>any brackets $180^\circ < x < 270^\circ$ ✓ not $180^\circ \in x \in 270^\circ$</i>	✓ critical values both correct ✓ notation <i>$180^\circ < x < 270^\circ$ 0 mark</i> (2)
6.3	$PQ = \cos 2x - (\sin x - 1)$ $= 1 - 2\sin^2 x - \sin x + 1$ $= -2\sin^2 x - \sin x + 2$ / $2\sin^2 x + \sin x - 2$ $\sin x = -\frac{b}{2a}$ let $\sin x = k$ $= \frac{-(-1)}{2(-2)}$ $k = \frac{-(-1)}{2(-2)}$ $\sin x = -\frac{1}{4}$ $\therefore x = 194,48^\circ$ or $x = 345,52^\circ$	✓ $PQ = \cos 2x - (\sin x - 1)$ Subtraction ✓ $\cos 2x = 1 - 2\sin^2 x$ ✓ substitution into formula <i>$-\frac{b}{2a}$ / k-method / complete / square / $dx = 0$</i> ✓ $\sin x = -\frac{1}{4}$ ✓ $194,48^\circ$ ✓ $345,52^\circ$ (6)

[10]

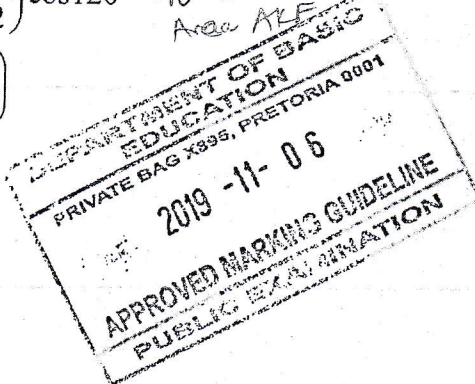
$\sin x - 1 = \cos 2x$
 $\sin x - 1 = 2\sin^2 x - 1$ max 1/6



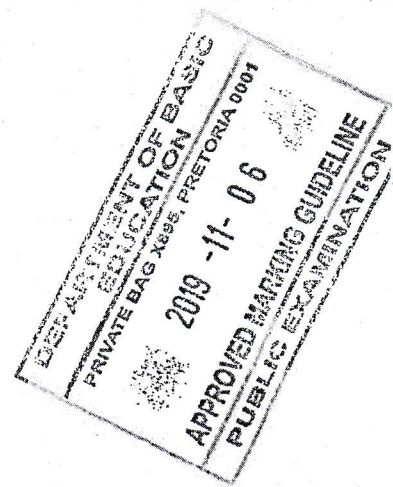
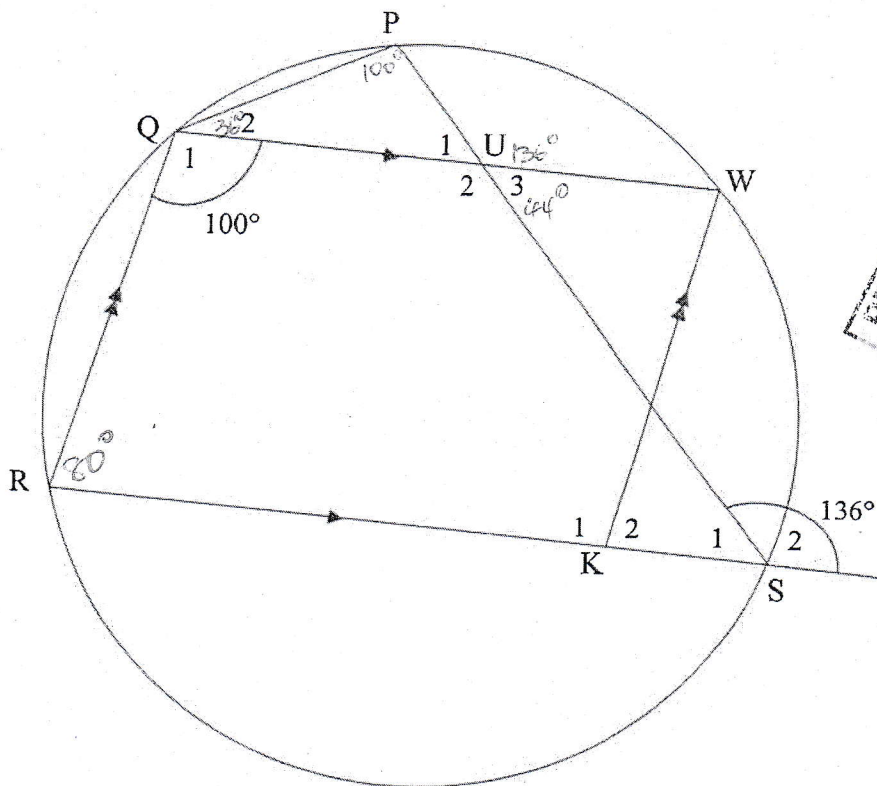
QUESTION/VRAAG 7



<p>7.1</p>	$\sin 60^\circ = \frac{AK}{x}$ <p>Accept $AK = \frac{x \sin 60^\circ}{\sin 90^\circ}$</p> $AK = x \sin 60^\circ \text{ or } \frac{\sqrt{3}}{2}x \text{ or } 0,866x$ <p><i>Pythagoras</i></p>	<p>✓ sine rule ✓ trig ratio ✓ answer</p> <p>(2)</p>
<p>7.2</p>	<p>$\hat{KCF} = 120^\circ$</p>	<p>✓ answer</p> <p>(1)</p>
<p>7.3</p>	$KF^2 = CF^2 + CK^2 - 2CF \cdot CK \cos \hat{KCF}$ <p>CA from 7.2</p> $= x^2 + \left(\frac{x}{2}\right)^2 - 2x\left(\frac{x}{2}\right) \cos 120^\circ$ <p>90° max 9/7 for Area AKF</p> $= x^2 + \frac{x^2}{4} - x^2\left(-\frac{1}{2}\right)$ $= \frac{7x^2}{4}$ $KF = \frac{\sqrt{7}x}{2}$ <p>$\hat{AKF} = y$</p> <p>Area $\Delta AKF = \frac{1}{2} \cdot AK \cdot KF \sin \hat{AKF}$</p> $= \frac{1}{2} \cdot \frac{\sqrt{3}x}{2} \cdot \frac{\sqrt{7}x}{2} \sin y$ $= \frac{x^2 \sqrt{21} \sin y}{8}$	<p>✓ correct use of cosine rule ✓ substitution ✓ $\cos 120^\circ = -\frac{1}{2}$ ✓ $KF = \frac{\sqrt{7}x}{2}$ ✓ correct use of area rule ✓ substitution ✓ answer in terms of x and y</p> <p>(7)</p>



QUESTION/VRAAG 8

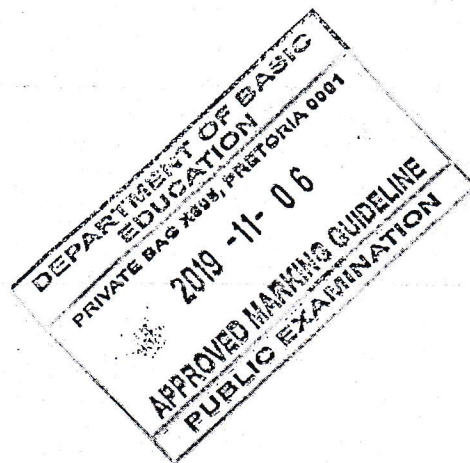


$\hat{Q}_1 + \hat{Q}_2$
 $\hat{Q}_1 + 2$

8.1.1	$\hat{R} = 80^\circ$ [co-int \angle s/ <u>ko-binne</u> \angle e; <u>QW RK</u>] penalise if not stated	✓S ✓R	(2)
8.1.2	$\hat{P} = 100^\circ$ [opp \angle s of cyclic quad/ <u>teenoorst</u> \angle e v <u>koordevh</u>] Kuh	✓S ✓R	(2)
8.1.3	$P\hat{Q}R = 136^\circ$ [ext \angle of cyclic quad/ <u>buite</u> \angle v <u>koordevh</u>] $\hat{Q}_2 = 36^\circ$ OR $P\hat{U}W = \hat{S}_2 = 136^\circ$ [corresp \angle s/ <u>ooreenkomstige</u> \angle e; <u>QW RK</u>] $P\hat{Q}W + \hat{P} = P\hat{U}W$ [ext \angle s of/ <u>buite</u> \angle van Δ QPU] $P\hat{Q}W + 100^\circ = 136^\circ$ $P\hat{Q}W = 36^\circ$ OR $\hat{U}_3 = 180^\circ - 136^\circ = 44^\circ$ [co-int \angle s/ <u>ko-binne</u> \angle e; <u>QW RK</u>] $\hat{U}_1 = \hat{U}_3 = 44^\circ$ [vert opp \angle s/ <u>regoorstaande</u> \angle e] $P\hat{Q}W = 180^\circ - (100 + 44^\circ)$ [sum of \angle s in Δ / <u>som</u> \angle e van Δ] $P\hat{Q}W = 36^\circ$	✓S ✓R ✓S ✓S ✓R ✓S	(3) (3)

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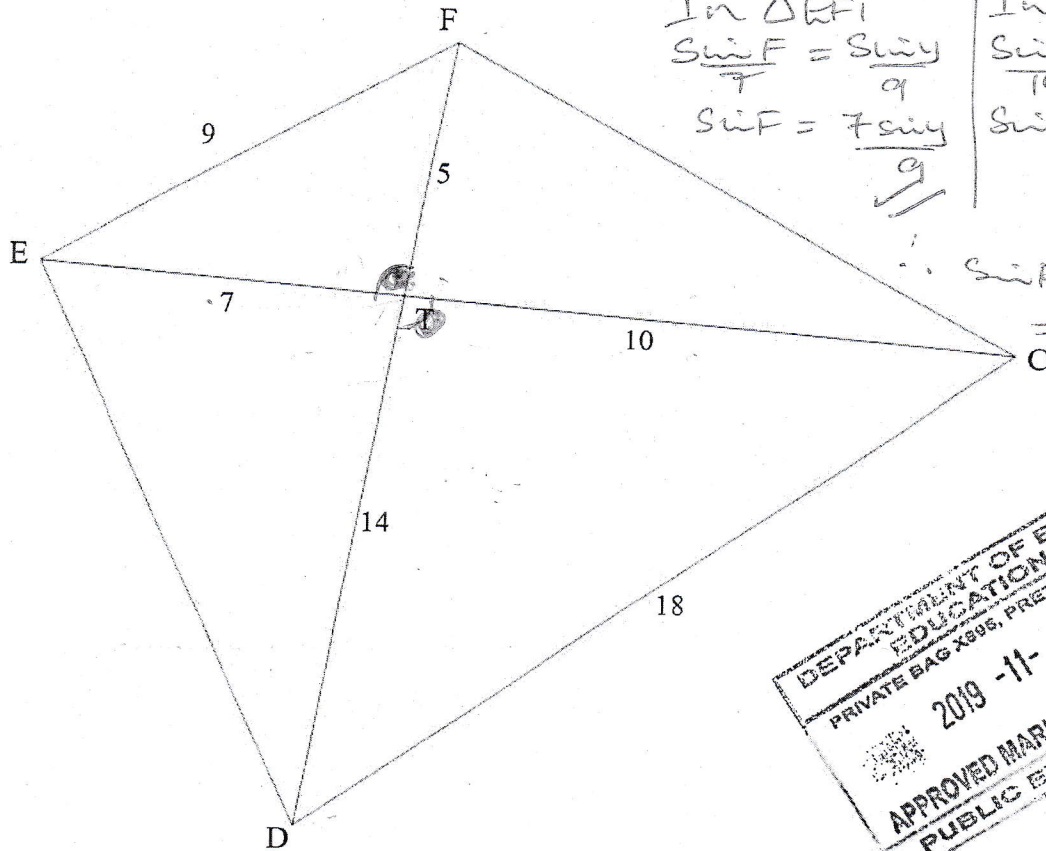
8.1.4	$\hat{U}_2 = \hat{S}_2 = 136^\circ$	[alt \angle s/verwiss \angle e ; QW RK]	✓S ✓R	(2)
	OR			
	$\hat{U}_2 = 100^\circ + 36^\circ$ $= 136^\circ$	[ext \angle s of/buite \angle van Δ QPU]	✓S ✓R	(2)
	OR			
	$\hat{U}_2 = \hat{P}\hat{U}W = 136^\circ$	[vert opp \angle s/regoorstaande \angle e]	✓S ✓R	(2)
	OR			
	$\hat{U}_2 = 180^\circ - \hat{U}_3$ $= 180^\circ - 44^\circ$ $= 136^\circ$	[\angle s on a str line/ \angle e op reguithyn]	✓S ✓R	(2)



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8.2



8.2.1

In $\triangle EFT$ and $\triangle DCT$:

$$\frac{EF}{CD} = \frac{9}{18} = \frac{1}{2}$$

$$\frac{FT}{TC} = \frac{5}{10} = \frac{1}{2}$$

$$\frac{ET}{TD} = \frac{7}{14} = \frac{1}{2}$$

OR

$$\frac{FT}{ET} = \frac{5}{7} \quad \text{OR} \quad \frac{ET}{TD} = \frac{7}{14} = \frac{1}{2}$$

$$\frac{TC}{TD} = \frac{10}{14} = \frac{5}{7} \quad \text{OR} \quad \frac{FT}{TC} = \frac{5}{10} = \frac{1}{2}$$

$\hat{F} = \hat{C}$ (vert-opp- \angle)

✓✓ all 3 ratios = $\frac{1}{2}$

$\therefore \triangle EFT \parallel \triangle DCT$ [Sides of Δ in prop/ sye van Δ in dieselfde verh]
 $\therefore \hat{F} = \hat{C}$ 2 sides in prop + included \angle

✓ $\triangle EFT \parallel \triangle DCT$ ✓ R

OR

In $\triangle EFT$:

$$49 = 25 + 81 - 2(5)(9)\cos\hat{F}$$

$$\cos\hat{F} = \frac{19}{30}$$

$$\hat{F} = 50,7^\circ$$

In $\triangle DTC$:

$$196 = 100 + 256 - 2(10)(18)\cos\hat{C}$$

$$\cos\hat{C} = \frac{19}{30}$$

$$\hat{C} = 50,7^\circ$$

✓✓ $\hat{F} = 50,7^\circ$

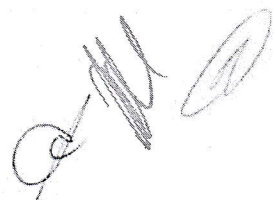
✓✓ $\hat{C} = 50,7^\circ$

(4)

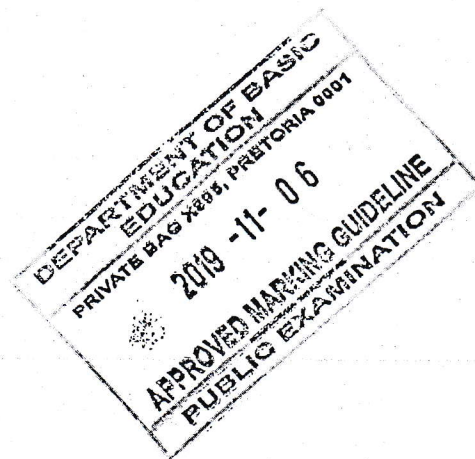
(4)

EFT \equiv DCT instead of \parallel max 2 marks

2 pairs of ratios but no 3rd pr } max 1/4
 or - angle }



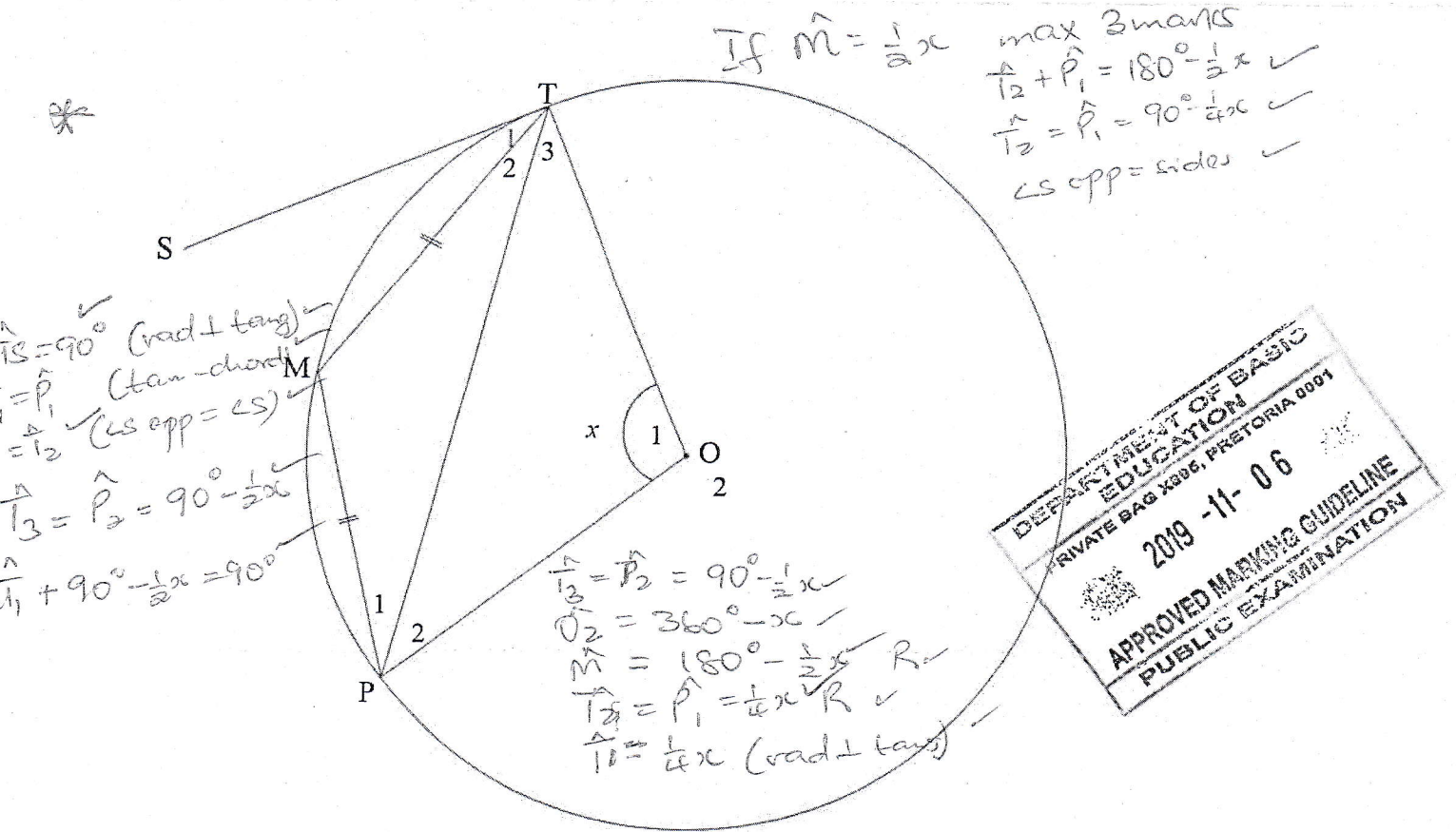
<p>8.2.2</p>	<p>$\hat{E}FD = \hat{E}CD$ [proved in 8.2.1] E, F, C and D are concyclic <u>EFCD is a cyclic quad</u> [converse \angles in the same segment/ <i>kuh</i> <i>omgekeerde \anglee in dies segment]</i> $\therefore \hat{D}FC = \hat{D}EC$ [\angles in the same segment/ <i>omgekeerde \anglee in dies segment]</i></p>	<p>\checkmarkS \checkmarkR \checkmarkR</p>
		<p>(3) [16]</p>



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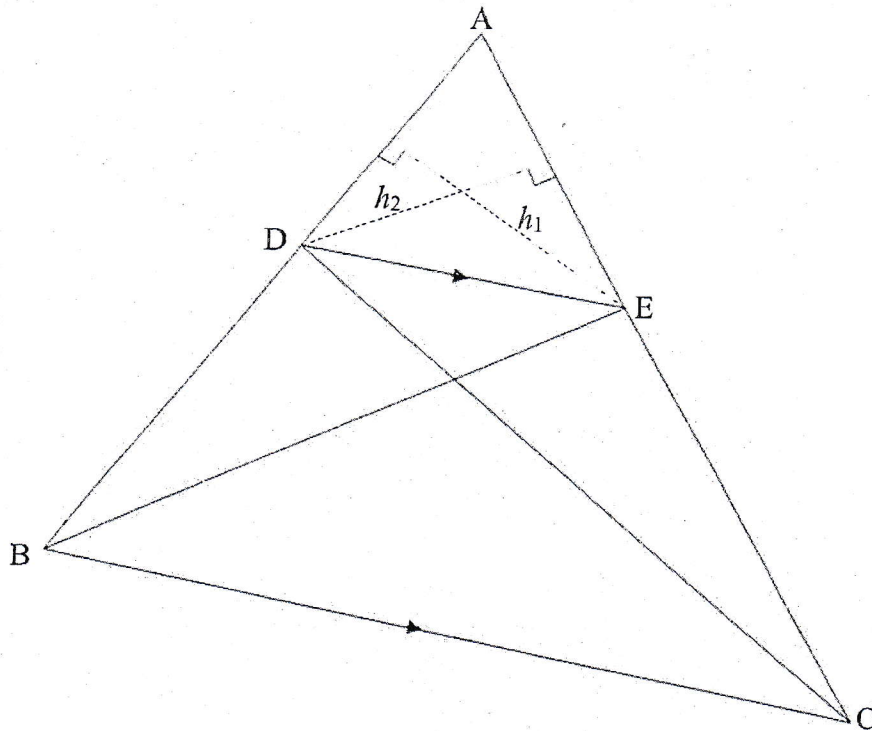
QUESTION/VRAAG 9



$\hat{O}_2 = 360^\circ - x$ [\angle s round a pt/ \angle e om 'n punt]	S	$\checkmark \hat{O}_2 = 360^\circ - x$
$\therefore \hat{M} = 180^\circ - \frac{1}{2}x$ [\angle at centre = $2 \times \angle$ at circumference/ middelpunts $\angle = 2 \times$ omtreks \angle]	S R	$\checkmark \hat{M} = 180^\circ - \frac{1}{2}x \checkmark R$
$\therefore \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ [sum of \angle s in Δ /som \angle e van Δ]	S	$\checkmark \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$
$\therefore \hat{T}_2 = \hat{P}_1 = \frac{1}{4}x$ [\angle s opp equal sides/ \angle e teenoor gelyke sye]	S R	$\checkmark \hat{P}_1 = \frac{1}{4}x \checkmark R$
$\therefore \hat{S}TM = \hat{P}_1 = \frac{1}{4}x$ [tan chord theorem/raaklyn koordstelling]	R	$\checkmark R$
(7)		
OR/OF		
$\hat{O}_2 = 360^\circ - x$ [\angle s round a pt/ \angle e om 'n punt]		$\checkmark \hat{O}_2 = 360^\circ - x$
$\therefore \hat{M} = \frac{1}{2}\hat{O}_2$ [\angle at centre = $2 \times \angle$ at circumference]		$\checkmark S \checkmark R$
$\therefore \hat{T}_2 + \hat{P}_1 = 180^\circ - \hat{M}$ [sum of \angle s in Δ /som \angle e van Δ]		$\checkmark S$
$\therefore \hat{T}_2 = \hat{P}_1$ [\angle s opp equal sides/ \angle e teenoor gelyke sye]		$\checkmark R$
$= \frac{180^\circ - \hat{M}}{2} = \frac{180^\circ - \frac{1}{2}\hat{O}_2}{2} = \frac{180^\circ - \frac{1}{2}(360^\circ - x)}{2} = \frac{1}{4}x$		$\checkmark S$
$\therefore \hat{S}TM = \frac{1}{4}x$ [tan chord theorem/raaklyn koordstelling]		$\checkmark R$
(7)		
[7]		

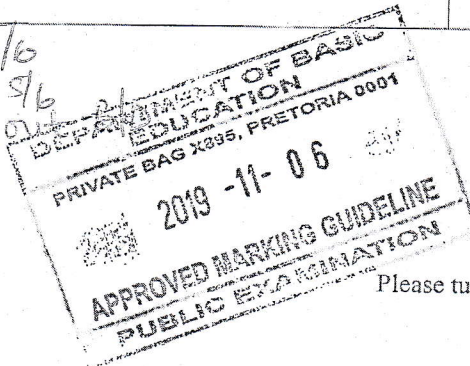
QUESTION/VRAAG 10

10.1



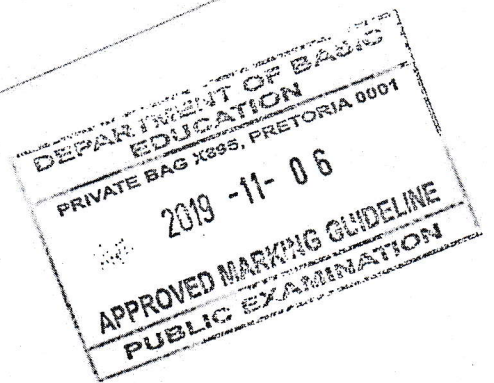
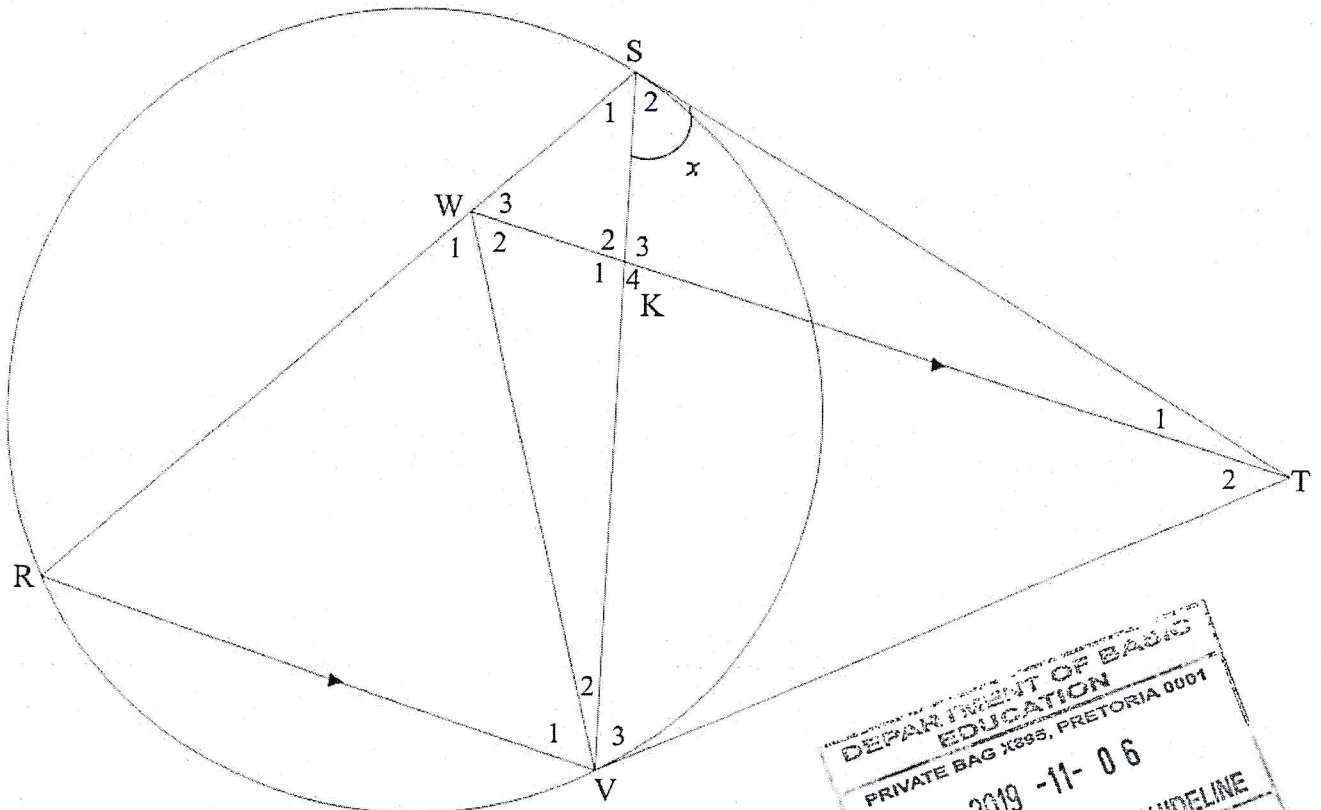
10.1	<p>Constr: Draw h_1 from E \perp AD and h_2 from D \perp AE Konstr: Trek h_1 vanaf E \perp AD en h_2 vanaf D \perp AE Constr. words/sketch/reason Proof/Bewys:</p> $\frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\frac{1}{2} AD \times h_1}{\frac{1}{2} DB \times h_1} = \frac{AD}{DB}$ $\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{\frac{1}{2} AE \times h_2}{\frac{1}{2} EC \times h_2} = \frac{AE}{EC}$ <p>But area $\triangle BDE = \text{area } \triangle DEC$ [same base & height or DE \parallel BC/ area $\triangle BDE = \text{area } \triangle DEC$ dies basis & hoogte; of DE \parallel BC] $\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$</p>	<p>✓ constr/konstr OR reason: common vertex or same height</p> $\frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\frac{1}{2} AD \times h_1}{\frac{1}{2} DB \times h_1}$ $\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{AE}{EC}$ <p>✓ S ✓ R ✓ S</p>
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No reason 5/6
 no construction 5/6
 one statement left



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10.2



<p>10.2.1</p>	<p>$\hat{V}_3 = x$ [Tans from same point/raaklyne vanaf dieselfde pt] $\hat{R} = x$ [tan chord theorem/raaklyn koordstelling] $\hat{W}_3 = x$ [corresp \angles/ooreenkomstige \anglee; $WT \parallel RV$] <i>$\hat{W}_2 = \hat{S}_2$ B/D \hat{W}_3 comes after \hat{R}</i></p>	<p>\checkmark S \checkmark R tangent key word \checkmark S \checkmark R \checkmark S \checkmark R</p>
<p>10.2.2(a)</p>	<p>$\hat{V}_3 = \hat{W}_3 = x$ [proved in 10.2.1] <i>provided they are mentioned in 10.2.1</i> W, S, T and V are concyclic/is konsiklies WSTV is a cyclic quad [<u>converse</u> \angles in the same segment/equal \angles on chord/ST <i>Omgekeerde \anglee in dieselfde segment</i>] <i>Assumed WSTV cyclic quad in 10.2.1 opmarks</i></p>	<p>\checkmark S \checkmark R <i>on chord/ST</i></p>
<p>10.2.2(b)</p>	<p>$\hat{W}_2 = \hat{S}_2 = x$ [\angles in the same segment/\anglee in dies segment] $\hat{V}_1 = \hat{W}_2 = x$ [alt \angles/verwiss \anglee; $WT \parallel RV$] But $\hat{R} = x$ [proved in 10.2.1] $\therefore \hat{R} = \hat{V}_1 = x$ $\therefore WR = WV$ [sides opp equal \angles/sye teenoor gelyke \anglee] ΔWRV is isosceles/is gelykbenig OR/OF</p>	<p>\checkmark S \checkmark R \checkmark S/R \checkmark S</p>

	$\hat{S}_2 = \hat{W}_2 = x$ [\angle s in the same segment] $\hat{W}_2 = \hat{W}_3 = x$ $\hat{W}_2 + \hat{W}_3 = \hat{R} + \hat{V}_1$ [ext \angle of Δ] $\therefore \hat{V}_1 = x = \hat{R}$ $\therefore WR = WV$ [sides opp equal \angle s/sye teenoor gelyke \angle e] ΔWRV is isosceles/is gelykbenig	✓ S ✓ R ✓ S/R ✓ S (4)
10.2.2(c)	In ΔWRV and/en ΔTSV $\hat{R} = \hat{S}_2 (= x)$ <i>not necessary</i> [proved OR tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\therefore \Delta WRV \parallel \Delta TSV$ [\angle, \angle, \angle] Accept $\hat{V} = \hat{V} = x$ if ΔWRV and ΔTSV are mentioned OR/OF If $\hat{V} = \hat{V} = x$ common \angle max 1/3 In ΔWRV and/en ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved OR tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\hat{W}_1 = \hat{S}\hat{T}\hat{V} = x$ [sum of \angle s in Δ/\angle e van Δ] $\therefore \Delta WRV \parallel \Delta TSV$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.2(d)	$\frac{RV}{SV} = \frac{WR}{TS}$ [$\Delta WRV \parallel \Delta TSV$] $\therefore WR \times SV = RV \times TS$ $\frac{WR}{SR} = \frac{KV}{SV}$ [prop theorem/eweredighst; $WT \parallel RV$] $\therefore WR \times SV = KV \times SR$ $\therefore RV \times TS = KV \times SR$ $\therefore \frac{RV}{SR} = \frac{KV}{TS}$ No penalty for changing order of Δ 's identified OR/OF In ΔRVS and/en ΔVKT $\hat{S}\hat{V}\hat{R} = \hat{K}_4$ [alt \angle s, $WT \parallel RV$] $\hat{S}\hat{R}\hat{V} = \hat{V}_3$ [proven] $\Delta RVS \parallel \Delta VKT$ [\angle, \angle, \angle] $\therefore \frac{RV}{SR} = \frac{KV}{VT}$ but $VT = ST$ [tans from same point] $\therefore \frac{RV}{SR} = \frac{KV}{TS}$	✓ correct ratios ✓ $\frac{WR}{SR} = \frac{KV}{SV}$ ✓ R ✓ equating $WR \times SV = RV \times TS$ ✓ identifying correct Δ s ✓ proving \parallel ✓ correct ratio ✓ S (4)

DEPARTMENT OF BASIC EDUCATION
 PRIVATE BAG 945, PRETORIA 0001
 2019-11-06
 APPROVED MARKING GUIDELINE
 PUBLIC EXAMINATION

[25]

TOTAL/TOTAAL: 150